Arts & Science
MATH 116 CALCULUS II

Final Examination

26th April 2018

Please put a check mark to indicate your LECTURE Section and Instructor:

_____ **02** John Martin   _____ **04** Qingde Yang   _____ **06** Leslie Walter

Please write your names and IDs _clearly_ and _legibly_ in _ink_:

Family Name: _____   First Name: _____

Student ID: _____   NSID: _____

[5]   1. Use Part 1 of the Fundamental Theorem of Calculus to find $g'(3)$ if $g(x) = \int_{\sqrt{x}}^{\pi/4} \theta \arctan \theta \, d\theta$.

In Questions #2–#6, evaluate the integrals.

[5] 2. $\int \dfrac{1}{1+e^x}\,dx$

[5] 3. $\int x^2 \ln x \, dx$

[5] 4. $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx$

[5]   5. $\int \dfrac{1}{1+\sin x}\,dx$ (Hint. Multiply the numerator and the denominator by some suitable expression.)

[5] 6. $\displaystyle\int \frac{3x^2 + x - 1}{x^3 + x^2}\, dx$

[5]   7. Use L'Hospital's Rule to find the limit: $\displaystyle\lim_{x\to\infty} \frac{x^2 + 10}{e^{x^2}}$

[5]   8. Use the Trapezoidal Rule with $n = 4$ subintervals of equal length to approximate the value of the

integral $\displaystyle\int_{-2}^{2} \frac{1}{1+x^2}\, dx.$

In Questions #9, #10 and #11, we shall consider the region R bounded by the curves $y = \sin x$, $0 \le x \le \frac{\pi}{2}$, $y = 1$, and $x = 0$.

[5] 9. Find the area A of the region R. Be sure to draw a labelled diagram.

[6] 10. Refer to the same region R given on page 10. Use the disk or washer method to find the volume V of the solid obtained by rotating the region R about x-axis.

[6] 11. Refer to the same region R given on page 10. Use the method of cylindrical shells to find the volume V of the solid obtained by rotating the region R about the y-axis.

[6] 12. Find the average value of the function f defined by $f(x) = \dfrac{x}{\sqrt{x+3}}$ on the interval $1 \le x \le 6$.

[6] 13. Explain why the following integral is improper. Is it convergent or divergent? If it is convergent, evaluate it.

$$\int_1^\infty \frac{1}{1+x^2}\, dx \,.$$

[6]   14. A thick cable, 60 ft long and weighing 180 lb, hangs from a pulley on a crane. Find the work done
         if the pulley winds up 20 ft of cable.

[6] 15. Find the length L of the curve $y = \ln(\cos x)$ where $0 \le x \le \frac{\pi}{3}$.

[6] 16. The curve $y = \sqrt{1 + e^x}$, $0 \le x \le 1$, is rotated about the x-axis. Find the area of the resulting surface.

[7] 17. **Newton's Law of Cooling** states that at a given time instant t, an object's *rate of cooling*, $\frac{dT}{dt}$, is proportional to the difference between the object's temperature T and its surrounding temperature T_S provided that this difference is not too large. This law also applies to the rate of warming.

When a cold drink is taken from a refrigerator, its temperature is $5°C$. After 30 minutes in a $20°C$ room, its temperature has increased to $10°C$. What is the temperature of the drink after 60 minutes? Assume Newton's Law of Cooling applies.

[6] 18. A curve passes through the point (0, 3) and has the property that the slope of the curve at every point P is four times the y-coordinate of P. What is the equation of the curve?